The Equity Imperative

Combining Risk Factors for Superior Returns

Over the years, academic research has well-documented the notion of compensated risk factors. In Northern Trust’s 2013 paper, “Understanding Factor Tilts,” we find that equity style factors such as value, small size and quality have persistently provided excess returns on an absolute and risk-adjusted basis. The origin of compensated risk factors traces to William Sharpe’s capital asset pricing model (CAPM), where expected return is a linear function of a single risk factor – equity beta.1 Fama and French expanded this concept by demonstrating that additional risk premiums exist, notably small size and high value.2

One aspect of the compensated risk factor research receiving less attention is how investors should combine various risk factors to achieve superior risk-adjusted returns. To illuminate that, we show that because these compensated risk factors are independent, combining them will provide a diversification benefit to investors. However, how you combine them is critical to the strategy’s success, so we show that multi-factor intersection portfolios provide superior results over simple combinations.
**EQUITY TILTS**

Considerable evidence in financial literature shows that equity tilts toward certain risk factors, such as small size, high value, low volatility, high quality, high dividend yield and high momentum, can over time earn excess returns above a capitalization-weighted benchmark. Each year, many newly designed risk factors are proposed; however, these almost always are simply variations of known compensated risk factors, reinforcing the theoretical and empirical justification for these known risk factors. It is also common now to review active management in the context of factor exposures, using this attribution to determine whether a manager has excess alpha (true skill) above and beyond its factor exposures. While factor investing seems here to stay, the bulk of existing research addresses only the compensation of individual risk factors and pays relatively little attention to how to achieve the best results from a comprehensive multi-factor-based investment strategy.

The benefits of these strategies are predicated on factors being both compensated and independent. If returns to small size and high value are independent and, hence, uncorrelated, then we can gain from factor diversification by combining size and value tilts into a single multi-factor strategy. This is the same logic used in asset allocation decisions – combining uncorrelated asset classes tends to reduce portfolio volatility – and is the central insight of Harry Markowitz’s modern portfolio theory.³

A 1997 paper by Mark Carhart demonstrates that momentum, in addition to value and size, is also compensated and that the three factors are individually independent and uncorrelated.⁴ Since then, other credible risk factors have emerged, including dividend yield, quality and low volatility. Research suggests that these risk factors are also compensated and uncorrelated.

**EQUITY RISK FACTORS ORIGINS**

The genesis of equity risk factors traces to the seminal work of Fama and French, who proposed that size and value, in addition to the beta factor proposed by Sharpe, are compensated. In this context, “compensated” means that investors are receiving higher risk-adjusted returns than expected using a CAPM framework. Put another way: You may take higher risk but are more than compensated for that risk with greater returns.

As a result, investors and asset managers alike have turned their attention to such strategies. While the investment community clearly understands the main message of Fama and French — that size and value factor tilts can generate excess returns — what remains somewhat less well-understood is Fama and French’s secondary message that size and value are not only compensated but, in fact, *independent* sources of excess return.

**EXHIBIT 1: BARRA FACTOR CORRELATIONS**

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
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<tbody>
<tr>
<td>Volatility</td>
<td>1.00</td>
<td></td>
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<tr>
<td>Momentum</td>
<td>(0.14)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Size</td>
<td>(0.05)</td>
<td>0.10</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growth</td>
<td>0.26</td>
<td>(0.09)</td>
<td>(0.15)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>0.10</td>
<td>(0.12)</td>
<td>(0.11)</td>
<td>(0.09)</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quality</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.14)</td>
<td>0.06</td>
<td>0.16</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Dividend Yield</td>
<td>(0.13)</td>
<td>(0.09)</td>
<td>0.17</td>
<td>(0.13)</td>
<td>0.14</td>
<td>(0.07)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*Source: Northern Trust Quantitative Research, MSCI Barra*
Exhibit 1 details seven recognized factor correlations computed using the risk factor returns defined by MSCI Barra and Northern Trust for the quality factor. All factor correlations tend to be very low (either positively or negatively), confirming that factors are independent. As a result, we would expect a multi-factor strategy that combines two or more risk factors would have a lower volatility than a strategy based on any individual risk factor. This reduction in risk holds considerable appeal for most equity investors.

While the benefits of multi-factor strategies are self-evident and largely uncontroversial, there is little agreement within the industry as to the best approach for gaining multi-factor exposure. To date, two primary avenues have emerged:

**Simple Combination of Single Factor Portfolios:** This approach forms individual portfolios targeting single risk factors and makes discrete allocations to each portfolio (linear combination). For example, an investor may put 50% of his funds into a portfolio targeting high quality stocks and 50% into a portfolio targeting high value stocks to obtain a multi-factor tilt toward high quality/high value.

**Multi-Factor Intersection Portfolio:** This approach constructs a single portfolio targeting stocks at the intersection of two or more risk factors. For example, an investor may use 100% of his funds to buy stocks for his portfolio that are simultaneously high quality and high value to obtain a multi-factor high quality/high value tilt.

Which approach is better? The answer depends on who you ask. Index providers are quick to advocate for the simple combination of products in their suite of single-factor indices. However, here we provide a conceptual model, a mathematical illustration and empirical evidence that the simple combination of single-factor portfolios is sub-optimal and that multi-factor intersection portfolios produce superior results.

**MODELING THE CONCEPT**

The goal of a factor tilt is, of course, to capture the excess return associated with a style factor. The objective of a multi-factor tilt is to capture the excess return of several factors and reduce volatility through factor diversification thus producing higher risk-adjusted returns. The question is whether a blended implementation of factor tilt portfolios is the best way to achieve investment objectives.

Exhibit 2 depicts a hypothetical universe of stocks such as the MSCI World or Russell 3000 index as a square arrayed along two dimensions: quality and value. In a simple combination implementation strategy, we would form a quality index hypothetically consisting of 50% of the names in the universe with the highest quality. The blue shaded region in Exhibit 2 depicts this index’s holdings. Similarly, we also would form a value index that also consists of 50% of the names in the universe with the highest value, which are shaded yellow in Exhibit 3.

There is little agreement within the industry as to the best approach for gaining multi-factor exposure.

Here we provide evidence that multi-factor portfolios produce superior results.
Importantly, since quality and value factors are independent, as our correlation matrix in Exhibit 1 shows, the overlap between these indices’ holdings will be approximately 50%. Thus, Exhibits 2 and 3 are reasonably accurate depictions of reality, despite their simplicity. Therefore, in our example, blending quality and value indices will force the investor to hold upward of 75% of the names in the universe. Independence guarantees that high-quality companies are a mix of high and low value and, conversely high-value companies are a mix of high and low quality. Thus, a blended strategy aimed at a quality/value tilt will hold many low-quality (tan region of Exhibit 3) and low-value (blue region of Exhibit 3) names, in this case up to half of the total portfolio. This negates the objectives of the multi-factor tilt.
In contrast, a multi-factor intersection portfolio targets only those names that are simultaneously high-quality and high-value, as the green shaded box in Exhibit 4 depicts. The intersection portfolio allows investors to hold a smaller subset of names and does not force them to buy low-quality and low-value stocks, as does the simple combination approach. As a result, we would expect the performance of the multi-factor intersection portfolio to exceed that of the simple combination approach, offering both higher returns and lower volatility.

Increasing the number of factors/indices exacerbates the problems of the simple combination approach. Assuming each risk factor index holds 50% of universe names and all risk factors are independent, then a three- and a four-factor tilt would require the investor to hold 87.5% and 93.8%, respectively, of the names in the universe. At these levels, the actual factor exposures are largely muted, and returns to the multi-factor strategy converge with the simple beta return of the universe. Hence, the simple combination of individual factor-tilt portfolios offers fleeting benefits.

**Empirical Evidence**

If our conceptual model is correct, then we would expect the empirical evidence to support the claim that multi-factor intersection portfolios outperform simple combinations or blends. To address this question, we created single-factor portfolios for size, value, volatility and dividend yield using the MSCI Barra definition of the factors and the Northern Trust Quality Score* (NTQS) for the quality. (NTQS is a proprietary method developed by Northern Trust that gauges multiple dimensions of quality grouped by management efficiency, profitability and cash flow.) Within the Russell 3000 universe, we ranked stocks each month from January 1979 to June 2013 based on these factors and placed stocks into five quintiles. Individual factor portfolios represent the equally weighted performance of the most advantageous two quintiles, namely the highest quality, highest value, smallest size, lowest volatility and highest dividend yield. Exhibit 5 shows the returns and volatilities of these portfolios, as well as 50/50 blends of individual factors with quality.

In a similar fashion, we created intersection portfolios using the same rankings and quintiles but selected only those stocks that were jointly within the top two quintiles of factor pairs. For example, for a high quality/high value intersection portfolio, we would include only those names that were simultaneously in both the top two quintiles of quality and the top two quintiles of value. Exhibit 5 shows the return and volatility metrics of these intersection portfolios.

*"Insights on Defining Quality," Northern Trust, November 2013.*
After calculating the annualized returns and standard deviations, we found that all of our individual factor portfolios outperformed the benchmark Russell 3000 index. This was expected, as substantial evidence shows these are compensated risk factors that earn excess returns over time. Note also that the blend portfolios have returns and volatilities that are generally at the midpoint between their two constituent portfolios, also as expected. Pairwise T-tests comparing the blend portfolios’ returns to the Russell 3000 benchmark all yield T-statistics well above the 99% confidence level, indicating there are statistically significant excess returns to blend portfolios.

*Significant at the 95% confidence level
**Significant at the 99% confidence level

Source: Northern Trust Quantitative Research
However, for all factor pairs the return of the intersection portfolio significantly exceeds that of the blend, and the standard deviation roughly equals or, for the volatility and dividend yield, is significantly less than that of the blend portfolio. For example, the high quality/high value blend earned an average return of 16.7% over the nearly 35-year period when the intersection portfolio returned 22.0% with approximately the same volatility. The high quality/low volatility intersection portfolio earned a 1.2% premium over the blend portfolio (16.9% versus 15.7%) but had considerably lower portfolio volatility of 13.7% versus 16.0% for the blend. Exhibit 6 illustrates the risk/return characteristics of the blend (green font) and intersection (black font) portfolios.

Pairwise T-tests comparing returns of intersection portfolios versus the Russell 3000 benchmark are all significant at the 99% confidence level. Tests were also performed comparing the returns of the blend and intersection portfolios. All these test statistics are significant at the 95% confidence level, indicating that intersection portfolios do, indeed, earn higher returns than the blended counterparts, as our conceptual model predicted. Exhibits 7 – 10 show the annualized rolling three-year returns of the blend and the intersection portfolios.
EXHIBIT 7: QUALITY AND SMALL SIZE

Full Period Performance (1/1979 – 6/2013) – Quality 16.8%, Small Size 13.7%, Blend 15.3%, Intersection 20.1%
Source: Northern Trust Quantitative Research

EXHIBIT 8: QUALITY AND HIGH VALUE

Full Period Performance (1/1979 – 6/2013) – Quality 16.8%, High Value 16.6%, Blend 16.7%, Intersection 22.0%
Source: Northern Trust Quantitative Research
Full Period Performance (1/1979 – 6/2013) – Quality 16.8%, Low Volatility 14.7%, Blend 15.7%, Intersection 16.9%
Source: Northern Trust Quantitative Research

Exhibit 9: Quality and Low Volatility

Exhibit 10: Quality and High Dividend Yield
SUPERIOR RESULTS
Multi-factor tilts implemented using a simple combination or blend of single factor portfolios are inefficient and sub-optimal from a risk/return perspective. They invest too much of the portfolio in names outside of the intersection portfolio. While the simple combination of individual factor portfolios is straightforward, it does not fully capture all the potential benefits of factor-based strategies. Our research shows that factor-based strategies investing at the intersection of compensated risk factors can produce superior risk-adjusted returns through higher returns, lower risk or both.

A FORMAL MODEL
In the appendix, we present a formal model of factor intersection and in doing so prove two key theorems regarding factor-based investment strategies. The first is that if more than one independent compensated risk factor exists, then no single factor portfolio is ever mean-variance efficient. The second illustrates the conditions necessary for an intersection portfolio to dominate a blend portfolio.

These models demonstrate that most intersection portfolios should be considered superior from a mean-variance perspective relative to their blended (simple combination) counterparts.

APPENDIX
Mathematical Illustration
While our conceptual model makes intuitive sense, it admittedly lacks precision. This is the cost of simplicity. However, it is easy to give a concrete mathematical illustration that the blending of single-factor indices is sub-optimal from a risk-and-return perspective. Our illustration will prove two important theorems concerning risk-factor investing.

Theorem 1: If more than one independent compensated risk factor exists, then no single factor portfolio is ever mean-variance efficient.

Proof
Assume two independent compensated risk factors such as value and quality exist. Consider a single-factor portfolio such as high value. If risk factors are independent, then constituents within this high-value portfolio must be evenly distributed between high- and low-quality stocks. We can, therefore, think of the high-value portfolio as being comprised of two sub-portfolios of roughly equal weighting, one consisting of high-quality stocks and one consisting of low-quality stocks.
In mean-variance analysis, a portfolio is considered efficient if it lies on the capital allocation line (CAL). As shown by Chen, Chung, Ho and Hsu, in the case of two sub-portfolios, the optimal weighting for each to obtain a combined portfolio that lies on the CAL is expressed as the solution to the following two equations:

**Equation 1**

\[ w_H = \frac{r_H \sigma^2_H - r_H \sigma_{HL}}{r_H \sigma^2_L + r_L \sigma^2_H - (r_H + r_L) \sigma_{HL}} \]

**Equation 2**

\[ w_L = 1 - w_H \]

\( w_H \) is the weight (0% to 100%) on the high-quality sub-portfolio

\( w_L \) is the weight on the low-quality sub-portfolio

\( r_H \) and \( \sigma^2_H \) are the expected excess return over a risk-free asset and the total excess return variance for the high-quality sub-portfolio

\( r_L \) and \( \sigma^2_L \) are the same for the low-quality sub-portfolio

\( \sigma_{HL} \) is the covariance between the high- and low-quality sub-portfolios

Since stocks are evenly distributed between high and low quality within the high-value portfolio, the implicit weightings on the two quality sub-portfolios is \( w_H = 50\% \) and \( w_L = 50\% \). However, we can construct the simple inequality that:

**Equation 3**

\[ w_H^* > 50\% \]

This says that the optimal allocation to the high-quality sub-portfolio \( w_H^* > 50\% \) if:

**Equation 4**

\[ \frac{r_H \sigma^2_H - r_H \sigma_{HL}}{r_H \sigma^2_L + r_L \sigma^2_H - (r_H + r_L) \sigma_{HL}} > 50\% \]

Rearranging Equation 4 we obtain:

**Equation 5**

\[ \frac{r_L}{\sigma^2_L - \sigma_{HL}} < \frac{r_H}{\sigma^2_H - \sigma_{HL}} \]

which states that the optimal allocation \( w_H^* > 50\% \) if, all else being equal, \( r_H > r_L \) or \( \sigma^2_H < \sigma^2_L \) or both.
In other words, if the excess return per unit of risk (Sharpe ratio) for the high-quality sub-portfolio is greater than that of the low-quality sub-portfolio, then the mean-variance optimal allocation to the high-quality sub-portfolio exceeds 50%. If this is true, then the implicit 50% weighting of high quality within the high-value portfolio is sub-optimal.

As Northern Trust’s paper, “Understanding Factor Tilts” highlights, the hallmark of a compensated risk factor is that stocks along the risk spectrum have different Sharpe ratios. For example, high-quality stocks tend to have both higher returns and lower volatilities than low-quality stocks; hence, the Sharpe ratio of high-quality stocks exceeds that of low-quality ones. The same is true for other risk factors such as value, size, volatility, quality, etc.

Since risk factors are defined by this Sharpe ratio spectrum, Equation 5 must necessarily be true. Hence, the optimal allocation to the high-quality sub-portfolio will exceed 50% and, thus, the value portfolio itself is mean-variance inefficient since it holds the high-quality sub-portfolio at a 50% weight. This completes the proof.

Value and quality were simply used as examples. If any two independent compensated risk factors exist, then the same results apply. Thus, we conclude that no single factor portfolio is ever mean-variance efficient.

**Theorem 2:** An intersection portfolio of risk factors will always mean-variance dominate a convex combination (blend) of risk factors if the ratio of the excess return of the intersection portfolio to the blend portfolio \( r_I / r_B \) exceeds the ratio of the variance of the intersection portfolio and the covariance of the intersection and blend portfolio \( (\sigma_I^2 / \sigma_{IB}) \), if \( (r_I / r_B) > (\sigma_I^2 / \sigma_{IB}) \).

**Proof:**

In this case, we define the intersection sub-portfolio \( I \) as mean-variance dominating the blend sub-portfolio \( B \) if the weight of sub-portfolio \( I \) along the CAL is 1 while the weight of the sub-portfolio \( B \) is zero. Under these conditions, we would never hold the blend sub-portfolio \( B \) and, hence, it is dominated by the intersection sub-portfolio \( I \).

We can rewrite Equation 4 as:

**Equation 6**

\[
\frac{r_I \sigma_B^2 - r_I \sigma_{IB}}{r_I \sigma_B^2 + r_B \sigma_I^2 - (r_I + r_B) \sigma_{IB}} > \alpha
\]

where \( \alpha = w^* \) is the optimal weight on the intersection sub-portfolio \( I \) and, by default, the optimal weight on the blended sub-portfolio is \( w_B^* = 1 - w_I^* \).
Rearranging Equation 6 we obtain:

**Equation 7**

\[
\frac{r_B}{(1 - \alpha)\sigma_B^2 - \alpha\sigma_m} < \frac{r_I}{\alpha\sigma_I^2 - (1 - \alpha)\sigma_m}
\]

If the intersection sub-portfolio \( I \) mean-variance dominates the blend sub-portfolio \( B \) then \( \alpha = w^*_I = 1 \) such that:

**Equation 8**

\[
\frac{r_B}{\sigma_m} < \frac{r_I}{\sigma_I^2}
\]

or

**Equation 9**

\[
\frac{\sigma_I^2}{\sigma_m} < \frac{r_I}{r_B}
\]

which completes the proof.

The implications of Equation 9 are significant, if not immediately obvious. Recall that the covariance of sub-portfolios \( I \) and \( B \) are defined as:

**Equation 10**

\[
\sigma_{m} = \frac{1}{n-1} \sum_{i=1}^{n} (r_I^i + r_B^i) (r_I^i + r_B^i)
\]

\( n \) is the number of observations
\( r_I^i \) is the return of the intersection portfolio in period
\( r_B^i \) is the return of the blend portfolio in period
\( r_I \) is the average return on the intersection portfolio
\( r_B \) is the average return on the blend portfolio

Recall also that the variance of the intersection sub-portfolio is defined as:

**Equation 11**

\[
\sigma_I^2 = \frac{1}{n-1} \sum_{i=1}^{n} (r_I^i - \bar{r}_I) (r_I^i - \bar{r}_I)
\]
Note that Equations 10 and 11 are quite similar. Specifically they are related by the equation:

**Equation 12**

\[ \sigma_{IB} = \rho_{IB} \sigma_i \sigma_B \]

which states that if the standard deviations of the intersection and blend sub-portfolios are similar and the correlation is high, then the covariance of the intersection and blend portfolios should approximately equal the variance of the intersection sub-portfolio.

We have strong reason to expect this to be the case. After all, both the intersection and blend sub-portfolios are trying to capture the same factor exposures, so we would expect the correlations between their return series to be relatively high. Similarly, we would anticipate the volatility of these two series to be at least equal (if not less in the intersection sub-portfolio). Hence, we expect:

**Equation 13**

\[ \sigma_{IB} \approx \sigma_i^2 \]

We can see from Exhibits 11 and 12 that this is, indeed, the case for the intersection and blend portfolios analyzed. Exhibit 12 shows a near-perfect one-to-one relationship between the variance of the intersection sub-portfolio and the covariance of the intersection and blend sub-portfolios. Further, the correlation between the returns of the two sub-portfolios is very high at greater than 0.97.

**EXHIBIT 11: PORTFOLIO VARIANCES, COVARIANCES AND CORRELATIONS**

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<tr>
<th>Factors</th>
<th>Variance Blend</th>
<th>Variance Intersection</th>
<th>Covariance</th>
<th>Correlation</th>
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<tr>
<td>Value/Quality</td>
<td>0.32%</td>
<td>0.32%</td>
<td>0.32%</td>
<td>0.99</td>
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<tr>
<td>Size/Quality</td>
<td>0.36%</td>
<td>0.37%</td>
<td>0.36%</td>
<td>0.99</td>
</tr>
<tr>
<td>Volatility/Quality</td>
<td>0.21%</td>
<td>0.16%</td>
<td>0.18%</td>
<td>0.97</td>
</tr>
<tr>
<td>Dividend Yield/Quality</td>
<td>0.24%</td>
<td>0.19%</td>
<td>0.21%</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Source: Northern Trust Quantitative Research

What does this mean? Returning to Equation 9, we see that if the covariance of the intersection and blend portfolios is approximately equal to the variance of the intersection portfolio, then the left-hand side of Equation 9 should approximately equal one. Thus, for the inequality to hold, the return on the intersection portfolio must be only slightly higher than the blend portfolio for the intersection sub-portfolio to dominate the blend portfolio on a mean-variance basis.
Exhibit 13 shows the actual ratios of intersection variance to covariance for the blend and intersection portfolios we analyzed. We can see that these ratios are approximately equal to one and, in the case of volatility and dividend yield, actually less than one. In these latter two cases, the intersection sub-portfolio could actually underperform the blend sub-portfolio on an absolute basis but still dominate the blend sub-portfolio on a risk-adjusted basis (lower return but even lower risk).

The last column of Exhibit 13 shows the ratio of returns on the intersection sub-portfolio and the blend sub-portfolio. Clearly, all return ratios are greater than their corresponding variance/covariance ratio such that Equation 9 holds, and we conclude that all intersection portfolios mean-variance dominate the corresponding blend portfolio. Thus, the blend portfolios would not be held in any amount in a mean-variance efficient allocation.

While these results are clearly specific to the examples given, we feel they are easy to generalize. We showed that intersection portfolios are more concentrated on risk factors and, thus, should earn higher returns if the risk factors are truly compensated. As a result, the ratio of $r_I / r_B$ should exceed one. In contrast, we argued that the ratio $\sigma_I / \sigma_B$ should generally equal one due to the overlap in objectives. As such, we would expect most intersection portfolios to mean-variance dominate their blended counterpart and, hence, blended portfolios (simple combination) should not be held in any amount.
FOOTNOTES

5. The benefits of multi-factor strategies are analyzed more extensively in Northern Trust’s “Understanding Factor Tilts” paper, June 2013

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